You've found Leonardo da Vinci's secret vault, secured by a series of combination locks. Fortunately, your treasure map has three codes:1210,3211000, and… hmm. The last one appears to be missing. Looks like you're gonna have to figure it out on your own. There's something those first two numbers have in common: they're what's called autobiographical numbers. This is a special type of number whose structure describes itself. Each of an autobiographical number's digits indicates how many times, the digits corresponding to that position occurs within the number.

The first digit indicates the quantity of zeroes, the second digit indicates the number of ones, the third digit the number of twos, and so on until the end. The last lock takes a 10-digit number, and it just so happens that there's exactly one ten-digit autobiographical number. What is it? Blindly trying different combinations would take forever. So, let's analyse the autobiographical numbers we already have to see what kinds of patterns we can find. By adding all the digits in 1210 together, we get 4 – the total number of digits.

This makes sense since each individual digit tells us the number of times a specific digit occurs within the total. So, the digits in our ten-digit autobiographical number must add up to ten. This tells us another important thing –the number can't have too many large digits. For example, if it included a 6 and a 7, then some digit would have to appear 6 time and another digit 7 times–making more than 10 digits. We can conclude that there can be no more than one digit greater than 5 in the entire sequence. So out of the four digits 6, 7, 8, and 9, only one – if any-- will make the cut. And there will be zeroes in the positions corresponding to the numbers that aren't used. So now we know that our number must contain at least three zeroes –which also means that the leading digit must be 3 or greater.

Now while this first digit counts the number of zeroes, every digit after it counts how many times a particular non-zero digit occurs. If we add together all the digits besides the first one –and remember, zeroes don't increase the sum –we get a count of how many non-zero digits appear in the sequence, including that leading digit. For example, if we try this with the first code, we get 2 plus 1 equal 3 digits. Now, if we subtract one, we have a count of how many non-zero digits there are after the first digit –two, in our example. Why go through all that? Well, we now know something important: the total quantity of non-zero digits that occur after the first digit is equal to the sum of these digits, minus one. And how can you get a distribution where the sum is exactly 1 greater than the number of non-zero positive integers being added together?

The only way is for one of the addends to be a 2, and the rest 1s. How many 1s? Turns out there can only be wo –any more would require additional digits like 3 or 4 to count them. So, now we have the leading digit of 3 or greater counting the zeroes, a 2 outing the 1s, and two 1s –one to count the 2s and another to count the leading digit. And speaking of that, it's time to find out what the leading digit is since we know that the 2 and the double 1s have a sum of 4, we can subtract that from 10 to get 6. Now it's just a matter of putting them all in place:6 zeroes,2 ones,1 two,0 threes,0 fours,0 fives,1 six,0 sevens,0 eights, and 0 nines. The safe swings open, and inside you find...Da Vinci's long-lost autobiography.